

FLS 6441 - Methods III: Explanation and Causation

Week 9 - Controlling for Confounding

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Classification of Research Designs

		Independence of Treatment Assignment	Researcher Controls Treatment Assignment?
Controlled Experiments	Field Experiments	✓	✓
	Survey and Lab Experiments	✓	✓
Natural Experiments	Natural Experiments	✓	
	Instrumental Variables	✓	
	Discontinuities	✓	
Observational Studies	Difference-in-Differences		
	Controlling for Confounding		
	Matching		
	Comparative Cases and Process Tracing		

Section 1

Controlling for Confounding

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- ▶ Or what if *everyone* is treated at the same point in time?
- ▶ We cannot use Difference-in-Differences
- ▶ For cross-sectional observational studies, the next-best alternative is...
- ▶ Controls!

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- ▶ **What we know:** Adding control variable X changes the comparison we are making:
 - ▶ **Treatment is *associated* with higher values of the Outcome...for units with the same values of X**

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- ▶ **What we don't yet know:** When does controlling allow us to say:
 - ▶ **Treatment *causes* higher values of the Outcome?**

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 - ▶ We have to make an argument and provide supporting evidence

Controlling for Confounding

- Why does controlling for confounders help provide conditional independence?

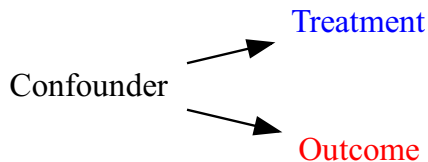
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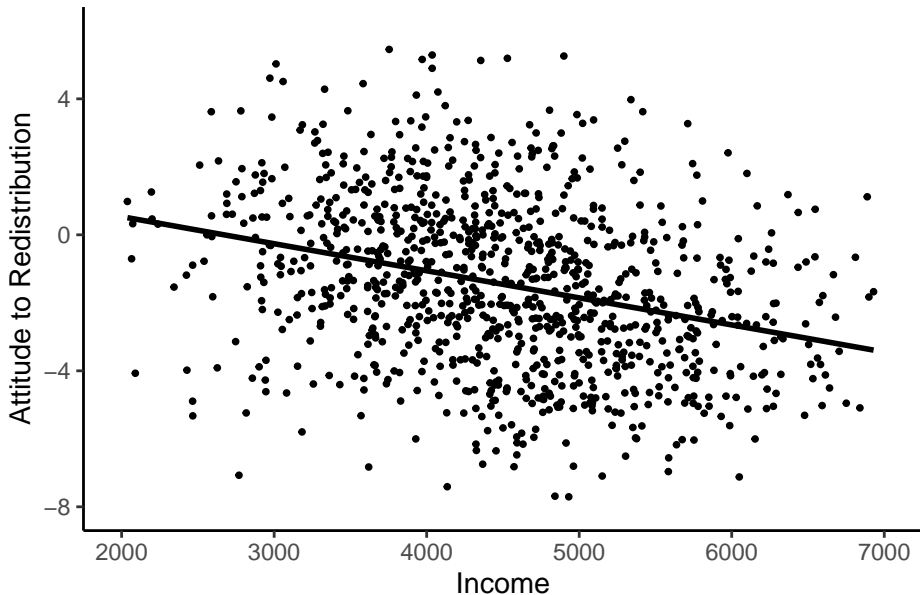
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- ▶ Why does controlling for confounders help provide conditional independence?
- ▶ We need to know what problem - what bias - confounders create:
 - ▶ The problem is of 'fake correlations' - D and Y look like they're related, even though treatment does not affect the outcome.
- ▶ Controlling *removes these fake correlations* by only comparing D and Y for units with the same value of X

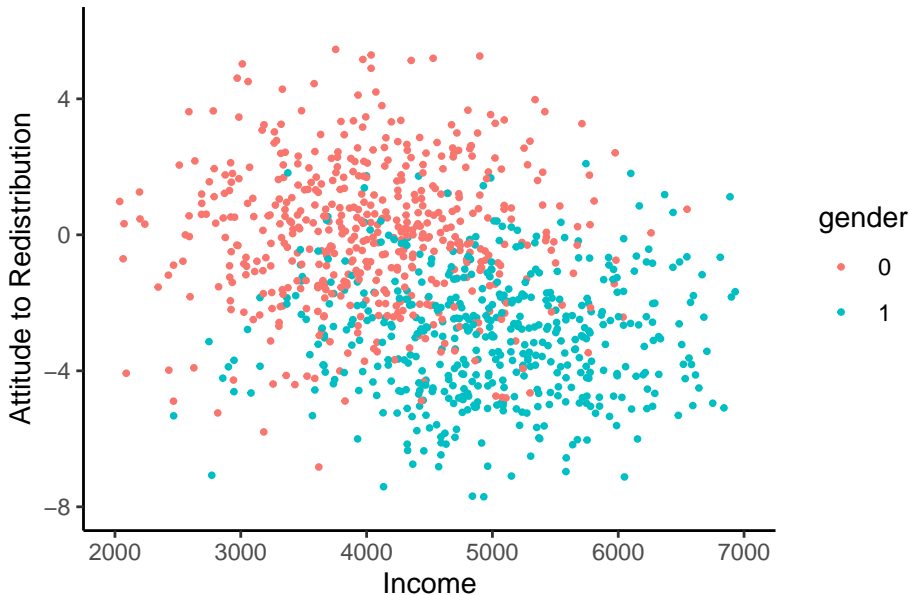
Causal Diagrams (DAGs)



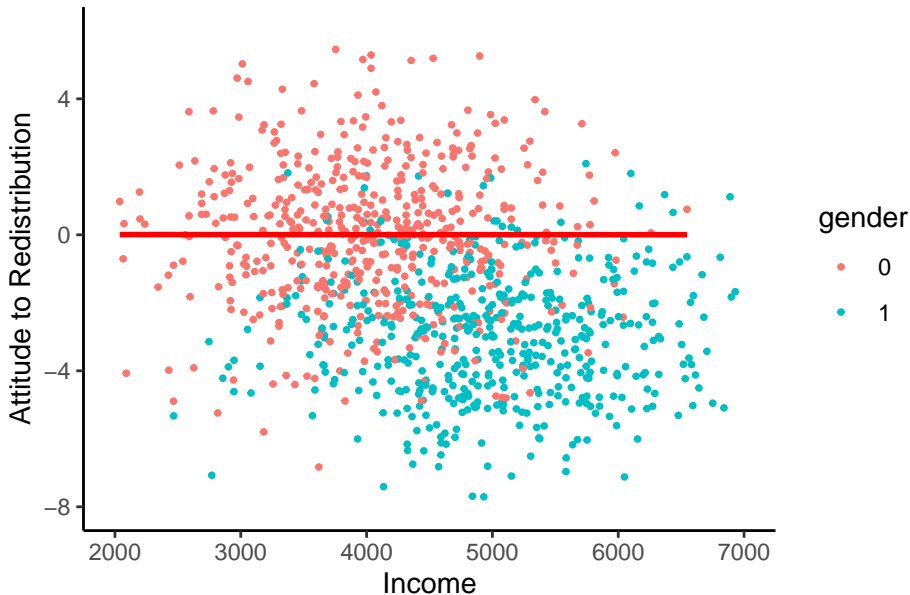
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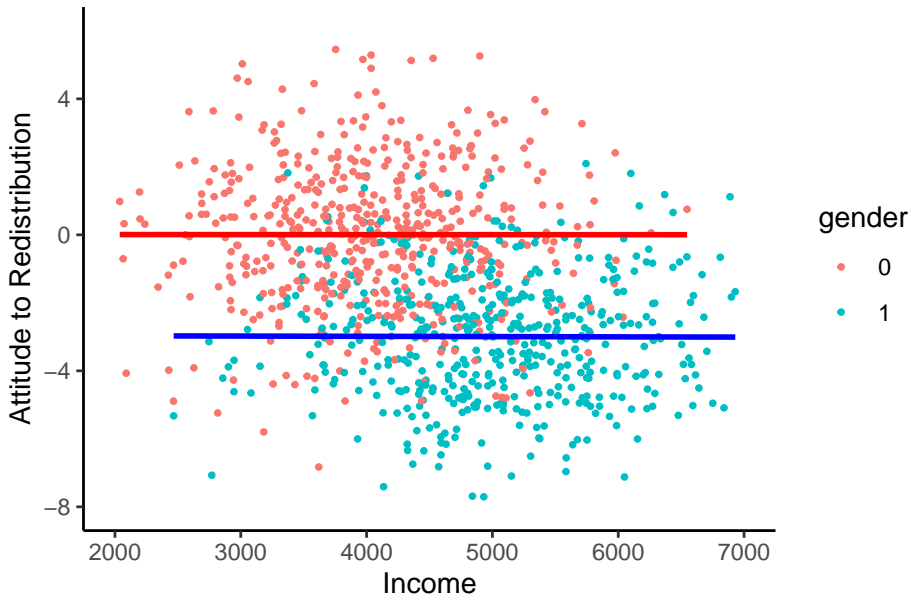
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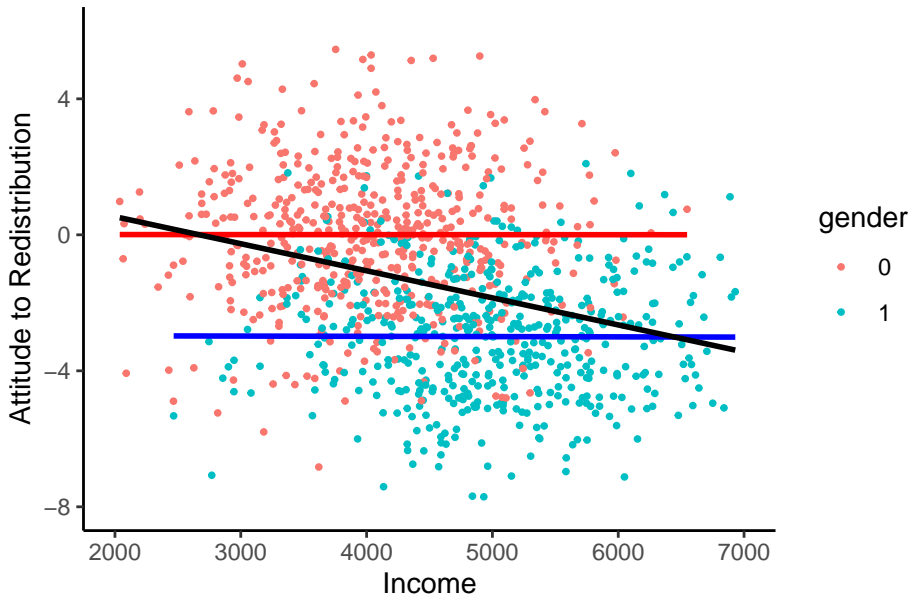
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$$\beta_{wrong} = \beta_{true} + \gamma\delta$$

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 - ▶ We have **created balance** between the treated and control groups on the confounder

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Section 2

Which Variables to Control For

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 - ▶ No circular loops!

Causal Diagrams (DAGs)

Treatment → Outcome

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 - ▶ We want to focus on one 'flow' of causation from treatment to outcomes

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 - ▶ We want to focus on one 'flow' of causation from treatment to outcomes
 - ▶ Avoiding mixing with the other flows of causation in the network

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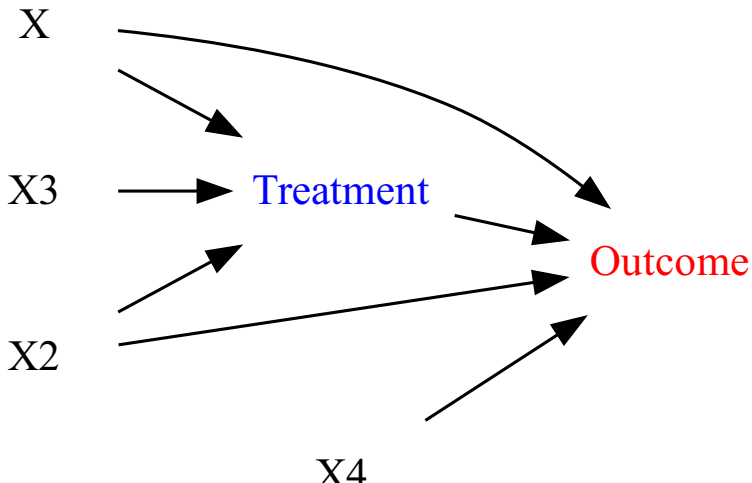
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 - ▶ Include these as control variables in our regression

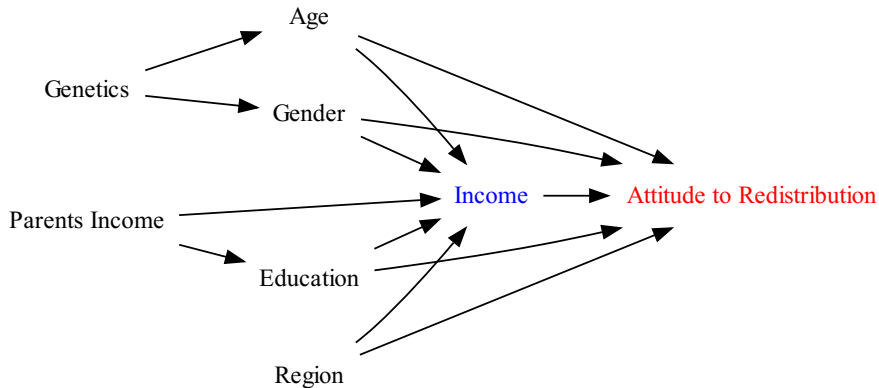
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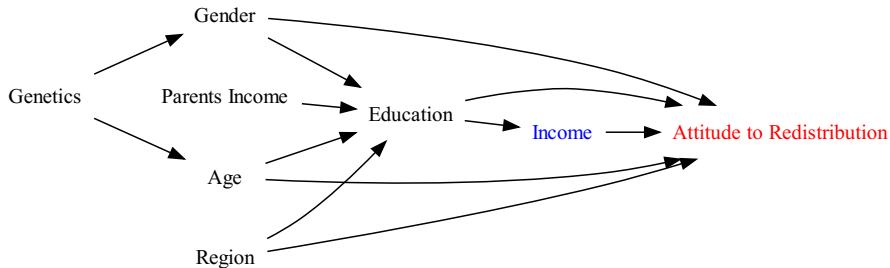
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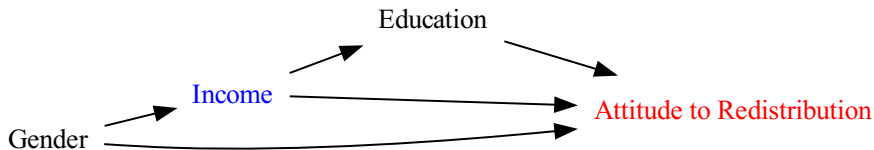
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 - ▶ Controlling for them changes the definition of the causal effect we are estimating

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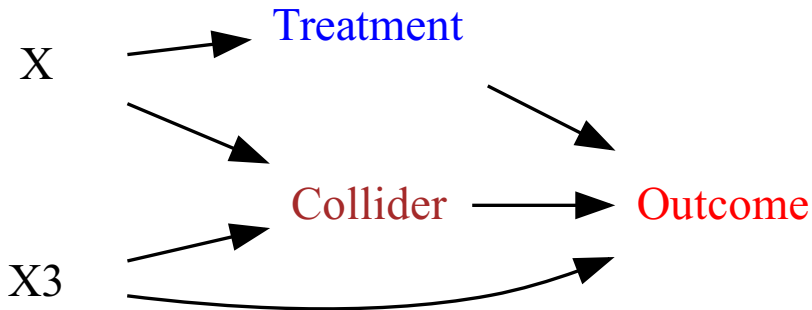
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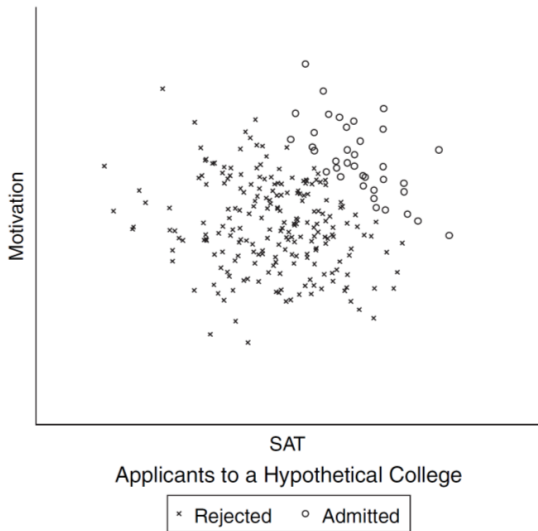
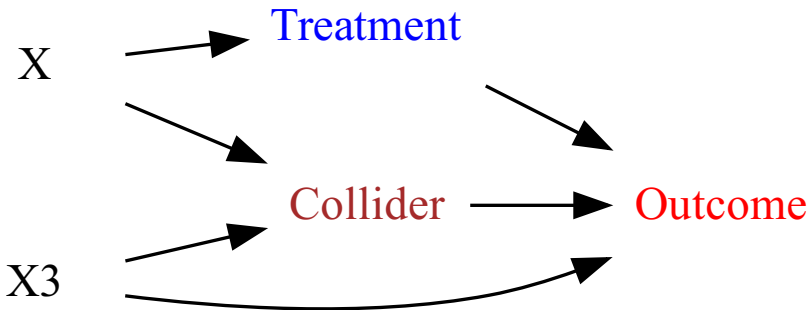


Figure 3.4: Simulation of conditional dependence within values of a collider variable.

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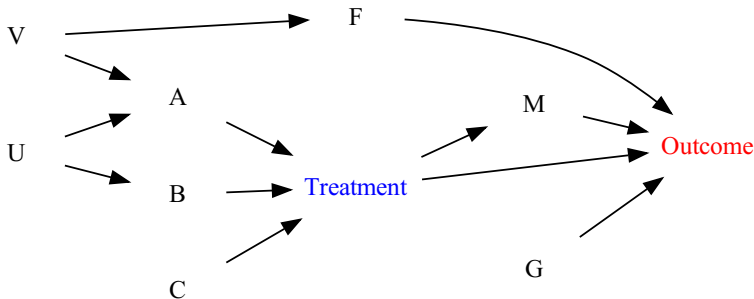
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5. Double-check your minimum set of control variables does not contain any post-treatment or collider variables



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